Energy-Switching Using Lévy Processes - An Application to Canadian and North American Data

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Abstract

The Paris agreement in 2016 marks a global effort to limit the increase in temperature. In that spirit, the Federal Government of Canada introduced a carbon tax to reduce greenhouse gas emissions. The main goal of this paper is to define the correct approach to carbon pricing. Following the method, introduce by Goutte and Chevalier (2015), we define the carbon price as the necessary tax to incite electricity producers to switch from coal to natural gas. The novelty of this paper is that we use this method for Alberta and North America. In addition, we consider the case of switching from natural gas to wind as a potential new approach to carbon pricing. After reviewing the two methods, we model prices under three stochastic procedures: Lévy Normal Inverse Gaussian (NIG), Lévy Normal and Heston model. Finally, we generalize our empirical technique to oil, natural gas and coal individually. The main finding of this article is that the Lévy NIG outperforms the Lévy Normal and Heston as it is able to take into account the jumpy and volatile nature of energy prices.

Key words: Energy Economics, Stochastic Processes, Carbon Pricing, Renewable Energies, Fossil Fuels, Fuel-Switching

1 Introduction

In October 2016, the federal government of Canada published "The pan-Canadian approach to pricing Carbon Pollution". The main goal of this act is to reduce greenhouse gas (GHG) emissions by taxing fossil fuels responsible for releasing carbon in the atmosphere. Over the years, fossil fuels, namely, coal, natural gas, and oil, have been seen as the main cause of temperature disruptions and extreme weather events. Climate specialists predict that temperatures could rise up to 5 degrees Celsius in 2100 (Chesney and Taschini, 2012). As fossil fuels contain carbon, once burnt, they allow energy to be generated, which in turn is important for health, education, political power and economic status (Sneideman, 2015). The Canadian consensus is in line with the climate agreement reached in Paris in 2016 where countries have agreed to a common effort to limit temperature increase to 1.5 degrees. Furthermore, developed countries are to provide help during extreme weather events and slow-onset such as the sea level rise. Finally, financial support should be given to developing countries in order to invest in clean energy (UNFCC, 2016).

If a consensus has been reached regarding efforts to protect the earth, the implementation of a carbon pricing scheme divides the fiercest economists, especially in Canada. Two systems are currently in place: cap and trade, and taxation. On the one hand, a cap and trade give an initial number of permits to emit CO2 to a company based on its activity. If an enterprise emits less than what the number of permits allows them, they can freely trade the number of permits in excess to another entity that wishes to pollute more. Economically more efficient, this approach gives an incentive to a company to reduce its pollution level in order to sell their permit. On the other hand, a taxation approach taxes every unit of carbon emitted at a fixed rate. As one can see, firms do not have an incentive to pollute less under the latter system. In Canada, Québec and Ontario adopted a cap and trade system, where the carbon price is $18 per tonne on average for the Québec Province (Tombe and Rivers, 2017). However, the western provinces of British Columbia and Alberta opted for a taxation system. The current carbon price in Alberta is $30 per tonne. The introduction of carbon pricing is said to have different impacts among regions. A case study of the province of Saskatchewan, whose economy is based on the extraction of natural resources, shows that GHG emissions are

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reduced but its economy is bound to shrink as the fuel-switching opportunities are rare (Liu et al, 2018). However, the BC example shows promising results. Introducing its carbon tax in 2008, Yamazaki (2017) found evidence that the system in place proved to be beneficial for the employment rate of the province.

This paper focuses on Alberta who introduced the carbon tax in 2017. Alberta is currently the biggest coal producer alongside British Columbia as they provide 85% of Canadian coal. The use of this fossil fuel currently generates 10% of the country's electricity. Moreover, the province supplies 71% of Canada's natural gas (NRCAN, 2018). In 2018, the carbon tax increased by 50% to reach $30 per tonne. Households electricity bills are expected to rise by $150 (Tombe, 2017). The main concern raised by economists is that the tax is going to affect poor households, and this, in turn, could lead to an increase in inequalities (Ambasta and Buonocore, 2018). Hence, a correct approach to carbon pricing is essential for the Albertan economy and its residents.

In this paper, we introduce an approach to carbon pricing based on the European energy market experience. Following the method introduced by Goutte and Chevallier (2015), we define the carbon price as the necessary price to incite companies to switch from coal to natural gas. Since the latter is less carbon intensive, this measure would considerably reduce GHG emissions. Besides, we also examine the case of switching from natural gas to wind. The economic content of this paper gives a detailed explanation of the fuel-switching and energy-switching processes. Moreover, we intend to generalize our statistical approach to the North American market and other energy indicators. In addition to the fuel-switching and energy-switching prices, we look at coal, natural gas, and oil individually. In order to model the prices, we consider three types of stochastic models: Lévy Normal Inverse Gaussian (NIG) process, Lévy Normal, and the Heston model.

The paper is structured in the following way. Section 2 provides the necessary economic background to understand how the fuel-switching price is defined. Section 3 presents the data used and gives the first insight into the Albertan energy market. Section 4 is concerned with the methodology and can be dissected into two parts: stochastic modeling and parameters estimation. Section 5 shows the empirical results found. Section 6 discusses the potential shortcomings of the paper and topics for further research and summarizes the main results.

2 Energy Economics and Energy-Switching

Energy markets depend on micro and macroeconomic factors and influence fuel-switching and energy-switching. The aim of this section is to first give the dynamic driving energy prices. Secondly, the notion of energy-switching is defined as well as the necessary conditions for it to happen and the potential problems arising from it. Thirdly, the carbon pricing formula is presented and reveals when companies have an incentive to pass from coal to natural gas (or natural gas to wind) and vice-versa based on current market conditions. Finally, we examine the factors influencing the price of wind.

Prices in the energy sector depend on political decisions and economic aspects. Competition among fossil fuel users and from alternative sources to generate energy, such as renewables, is a key factor in defining prices. Indeed, energy markets are often controlled by a monopolist who has sufficient power to dictate prices. Moreover, subsidies given by governments to clean-technologies can play an important role in the competitiveness of the industry. In addition to the competition facet, national allocation plans, which covers the initial number of permits (CO_2 allowances) and a penalty level, are identified as the main cause of price jumps. Furthermore, the volatility of the price of fossil fuels is also an element to take into consideration. In fact, coal prices are generally more stable than natural gas prices and, consequently, are more attractive for a company looking to reduce risks. Other variables potentially influencing prices are weather conditions and economic growth (Sjim et al, 2006; Seifert et al, 2008; Carmona et al, 2009).

Another influence omitted from the list above is fuel-switching, which represents the possibility to pass from a coal-fired plant to a natural gas plant, and vice-versa. Coal is generally cheaper and thus preferred by companies, even though it emits more CO_2. Therefore, in order for a switch to happen two conditions must be met. First, the carbon price (tax or current permit price) must be high enough and natural gas price low enough. Since natural gas emits less CO_2, a high carbon price favors its use. Second, there has to be the physical possibility to switch. During the winter season, the demand for electricity is typically higher than in the summer, and it is not unlikely that all plants are working at their maximum capacity, regardless of the type of fossil fuels used (Delarue, D'haeseleer, 2007). The fuel-switching process is a good start to model the carbon price since traditional abatement measures tend to invite producers to use cleaner energy than coal. However, as noted by Chesney and Taschini (2012), fuel purchasers tend to sign contracts with a long maturity and this impedes the fuel-switching process to be fully flexible. Consequently, this paper chooses to consider weekly fuel prices, rather than daily.

Electricity prices depend on the physical capacity to generate power, the presence of potential substitute and other economic factors. Therefore, an adequate formula must take into account the various aspects mentioned so far. This paper follows the method defined by Chevallier and Goutte (2015). Prices are defined by the marginal generation of technology and are expressed as the ratio between the fuel cost, FC, and the plant efficiency, η, which represents the necessary amount of energy needed to produce electricity:

$$MC = \frac{FC}{\eta}$$

Introducing a price on carbon requires the equation above to be revised. Considering that fossil fuels have a different impact on the environment, an emission factor, EF, based on C02 intensity is added as well as an emission cost, EC, per unit of carbon emitted. The revised formula reads:

$$MC = \frac{FC}{\eta} + \frac{EF}{\eta} EC$$

Fuel-switching occurs if the use of one fossil fuel to generate energy is cheaper than the other option. Therefore, by equalizing the marginal costs, one can define the minimum carbon price necessary for a switch to occur. Indeed, the only factor which is common to both fossil fuels is EC.

$$EC_{switch} = \frac{\eta_{coal} FC_{gas} - \eta_{gas} FC_{coal}}{\eta_{gas} EF_{coal} - \eta_{coal} EF_{gas}}$$

If the carbon tax defined by the Albertan government is lower than this price, then coal plants are said to be more profitable than gas plants.

The idea behind energy-switching based on wind follows the same formula. However, the factors influencing wind prices are quite different. Indeed, the use of wind depends on seasonal factors and its price depends on technological aspects, such as electricity storage. The Canadian Wind Energy Association (CanWea) provides further explanation regarding this topic. Moreover, wind is discussed in greater details in the next section.

3 Data

The data used in this project were gathered from various sources, such as NRG Stream, Bloomberg, Energy Information Association (EIA), Market Insider, Jem Energy, the city of Winnipeg and CanWea. This section provides insight into the distribution and evolution of energy prices over time, as well as the justification for the time period considered.

Following the formula of the fuel-switching (energy-switching) prices from the previous section, the different variables employed were retrieved from various data sources. When fuel-switching is considered, we need data regarding the fuel cost, efficiency parameters and emission factors as mentioned in the previous section. Prices of coal (in $/tonne) and prices of natural gas (in $/MMbtu) were retrieved from NRG Stream (Alberta data), EIA (natural gas North America data) and Market Insider (coal North America data). The emission factor component, EF, is expressed in kgCO2eq/MWhp. Financial data from the city of Winnipeg show that EF is equal to 210 for natural gas and 320 for coal. Similar to previous findings, coal is more harmful to the environment than natural gas. A 2004 study from JEM Energy calculated the efficiency of coal and natural gas plants in Alberta. The average efficiency for a coal and natural gas plant is respectively 32.6% and 31.1%. (Also 32% and 43% respectively in North America case, according to EIA.)

In order to compute the EC price, the unit of the data were changed from $/MMbtu and $/tonne to $/Mwh. Furthermore, weekly data were chosen as opposed to monthly data. Fuel and energy-switching are technically more likely to happen on a monthly basis, however, since we failed to obtain large time series data for the energy markets, we opted for a weekly approach. The next figures below (Fig. 1 and Fig. 2), represent the evolution of coal, natural gas, oil and fuel switching for both the Albertan and North American markets.

The results for the North American market were obtained using Bloomberg (oil), Market Insider (coal) and EIA (Henry Hub natural gas). In recent years, the prices for coal and natural gas have became extremely close to natural gas being even cheaper at times. Consequently, it is not surprising that we observe the fuel-switching price to have gone negative. The main implication is that pricing carbon using fuel-switching is not appropriate anymore. Regarding oil, we note that the financial crisis may have triggered a high-spike in its price. By looking at the three figures, we conclude that energy prices are characterized by high-spike and quick mean reversion, which justify our approach to use pure jumps methods. Additionally, it is apparent that prices go through periods of calm and stress, this, in turn, could imply that a stochastic volatility model, such as Heston model, yields better results than a classic geometric Brownian motion. Finally, it is indisputable that fuel-switching has become obsolete, therefore, we decide to model carbon under our energy-switching approach.

In recent years, renewable energies, wind in particular, have become an important source of electricity generation. A 2016 study by CanWea showed that wind power accounts for 50% of Denmark's electricity generation system and was the largest source of new Energy in Canada. The emission factor, associate to wind is equal to zero and it's efficiency ranges from 32-37% according to the EIA. Moreover, the cost of Wind is estimated to lie somewhere between 37.5$ and 42.5$ per MWh. Since data regarding wind cost is difficult to estimate, we generated a random uniformly distributed process to obtain the price of wind. The next figure shows the necessary price to switch from natural gas to wind. If the carbon

Fig. 1. Alberta Market. Prices are expressed in CAD/MWh.

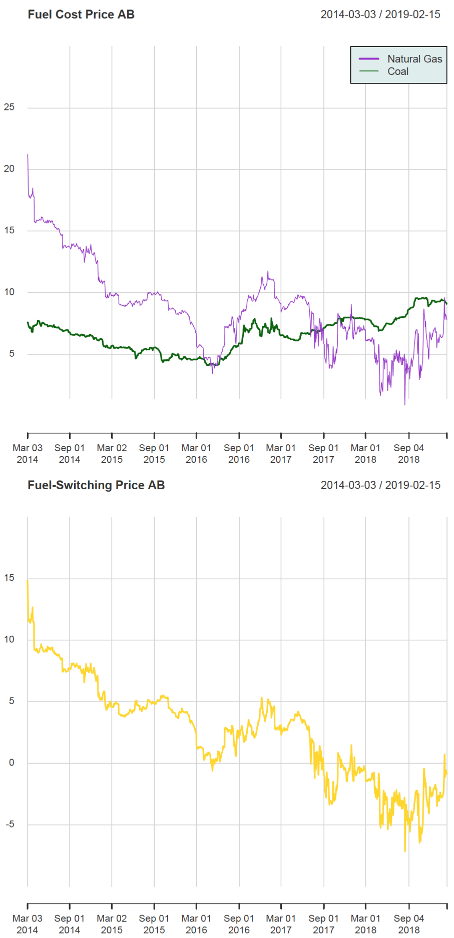
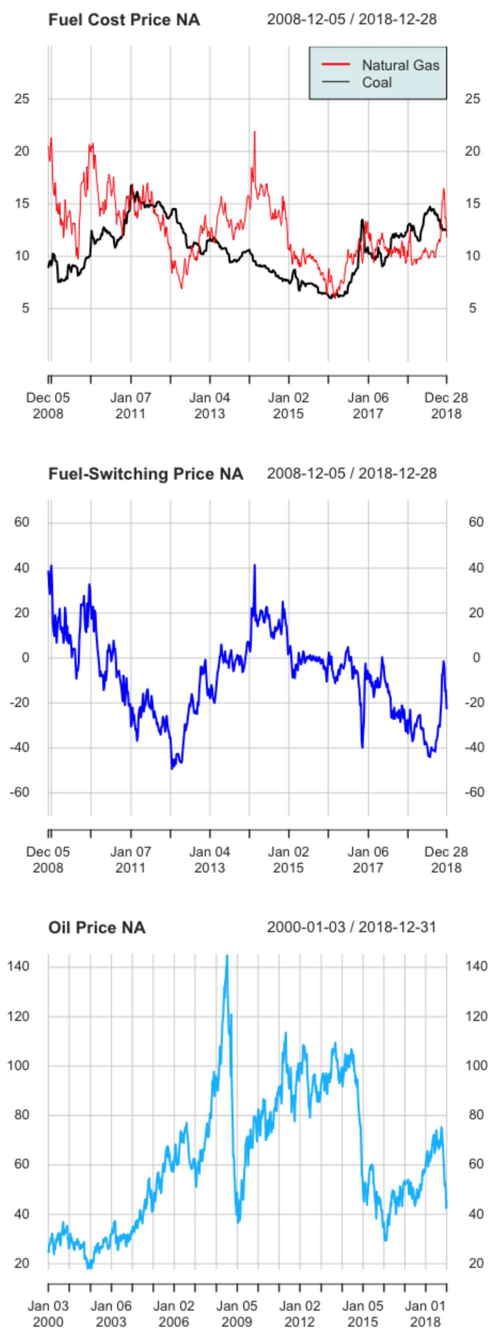


Fig. 2. North American Market. Prices are expressed in USD/MWh for the natural gas, coal and the fuel-switching price. The price of oil is given in USD/barrel.



tax is under the line, then natural gas plants are said to be more efficient than wind plants. The energy-switching price is quite high and reflects the current low price of natural gas in Alberta.

So far, our results have indicated that fuel-switching, considering the large drop in the natural gas price, is no longer an adequate approach. Moreover, the prices are characterized by jumps and mean-reversion and this confirms what has previously been observed in the literature. Therefore, the last step prior to stochastic modeling is to determine the empirical distribution of our data.

The distribution of returns is a good indicator of the problems that can arise when simulating prices under a geometric Brownian approach. Financial data, as illustrated by oil and energy-switching, depart from a Normal distribution. Indeed, extreme returns are more likely to happen than what the Normal predicts. Moreover, we notice that returns are generally skewed. Hence, the use of a NIG may be a more appropriate approach when simulating energy prices.

4 The stochastic Model

In this part, we are considering a panel of continuous mean reverting stochastic processes and the Heston model. The estimation methodology of parameters in mean-reverting stochastic processes is inspired from Chevalier and Goutte (2015).

Fig. 3. Energy Switching based on wind for Alberta. Prices are expressed in CAD/MWh.

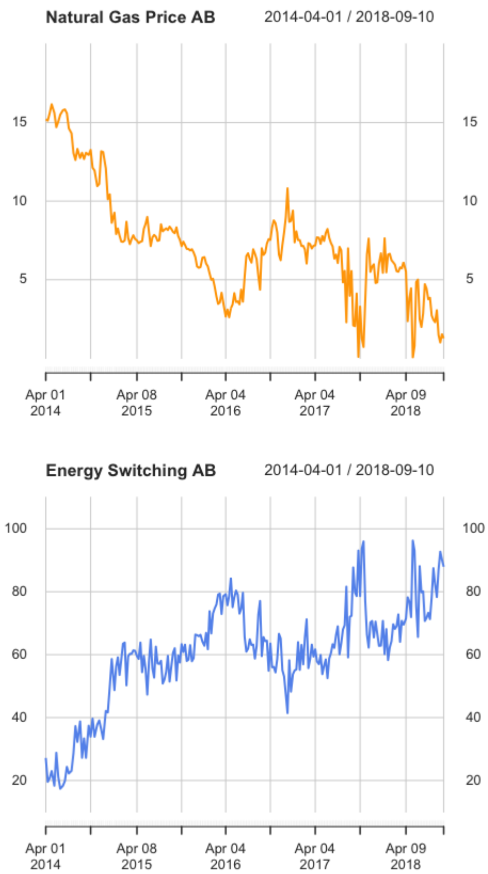
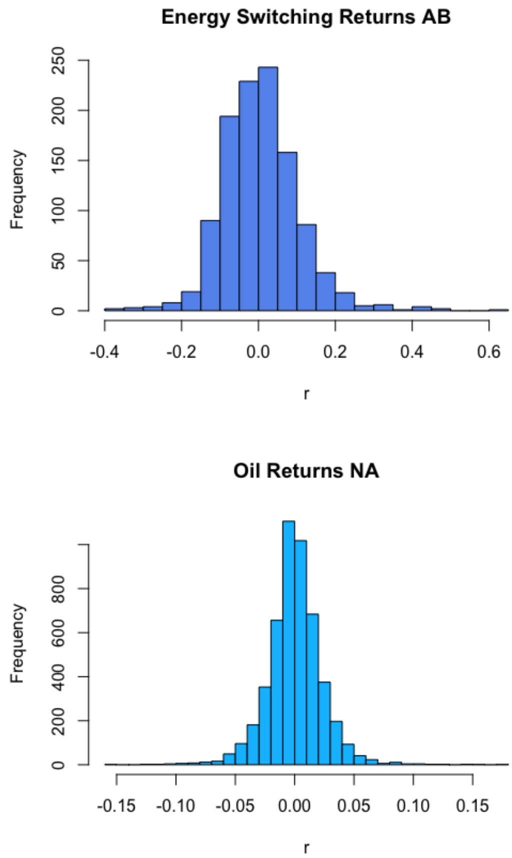


Fig. 4. Empirical Distribution of Returns



Let (Ω, F, P) be a probability space and in this paper, we assume all the stochastic models are under this probability space. We now consider three models: a continuous process with a Brownian motion, Lévy-driven Ornstein-Uhlenbeck process and Heston model. Then we fit each of them to our energy and fuel switching price and compare the result we have in the next part.

4.1 Mean-reverting process

To introduce two mean-reverting processes, we first need the definition of Lévy process. This section and the mathematical propositions that follow are inspired from Chevalier and Goutte (2015).

Definition 1: A Lévy process $\{X_t\}_{t\geq 0}$ is a stochastic process that it satisfies following properties:
1.$X_0 = 0$.
2.For any $s > 0$ and $t > 0$, we have that $X_{t+s} - X_t$ has the same distribution with X_s. i.e. It has stationary increments.
3. For $0 \leq t_0 < t_1 < ... < t_n$, $X_{t_i} - X_{t_{i-1}}$ are independent for all i. i.e. It has independent increments.
4. The path of a Lévy process are right continuous and admit left limit. i.e. X_t has càdlàg path.

One can treat the Lévy process as a combination of continuous process and discontinuous process. So two simple example of Lévy process will be Brownian Motion and Poisson Process. Next, we will give two mean reverting process, one is a continuous process with a browian motion and the other one is Lévy-driven Ornstein-Uhlenbeck processes.

Definition 2: Let $t \in [0, T]$, (X_t) be the solution to a stochastic differential equation:

$$dX_t = \kappa(\theta - X_t)dt + \sigma dY_t$$

with parameters κ,θ in $\mathbb{R}$, $\sigma \in \mathbb{R}^+$ and Y_t is another stochastic process.

If the Y_t is a standard Brownian motion then (X_t) is called continuous Ornstein-Uhlenbeck process.

Otherwise if Y_t is a Lévy process, we call (X_t) as a Lévy-driven Ornstein-Uhlenbeck process.

Remark 1: In this model, κ denotes the mean-reverting rate, θ denotes the long-run mean and σ denotes the volatility.

Remark 2: The Lévy process L can follow a panel of distributions for example the Variance Gamma distribution and Normal Inverse Gaussian distribution. In this paper we assume that it follows a Normal Inverse Gaussian(NIG) distribution, which has the probaility density function

$$\frac{\alpha\delta}{\pi} exp(\delta\sqrt{\alpha^2-\beta^2}+\beta(x-\mu))\frac{K_1(\alpha\delta\sqrt{1+(x-\mu)^2/\delta^2})}{\sqrt{1+(x-\mu)^2/\delta^2}}$$

where $\delta > 0$,$\alpha \geq 0$, $\gamma \geq 0$ and K_v is a Bessel function of the third kind with index v. This family of distribuiton was introduced by Barndorff-Nielsen(1998) and it is a continuous probability distribution that is defined as the Normal variance-mean mixture where the mixing density is the inverse Gaussian distribution.

Now we already have the model we want to fit, but we need to estimate the parameters in it based on the data we have. Next section gives an introduction to a parameter estimation method.

4.2 Parameter Estimations of mean reverting stochastic process

In this section we consider a two step parameter estimation method for the parameters in Lévy-driven Ornstein Uhlenbeck process. In Chevalier and Goutte (2015), they developed a least square method which minimize the empirical variance to obtain the parameter for Brownian motion and they used a constrained maximum likelihood method for estimating the NIG random variable.

Before estimating parameters, we first need to discretization the model. In practice, we observe the price at fixed times $0 = t_0 < t_1 < ... < t_n = T$, with $\Delta t = t_{k+1} - t_k$ constant. Thus we can first solve the stochastic differential equation and discretize the solution by:

$$X_{t_{k+1}} = X_{t_k} e^{-\kappa\Delta t} + \int_{t_k}^{t_{k+1}} \kappa\theta e^{-\kappa(t_{k+1}-s)} ds + \int_{t_k}^{t_{k+1}} \sigma e^{-\kappa(t_{k+1}-s)} dL_s$$

Rearrange the solution, we obtain:

$$X_{t_{k+1}} - X_{t_k} = m - aX_{t_k} + s\epsilon_{t_k}$$

with $m = (1 - e^{-\kappa\Delta t})\theta$, $a = 1 - e^{-\kappa\Delta t}$ and $s\epsilon_k = \int_{t_k}^{t_{k+1}} \sigma e^{-\kappa(t_{k+1}-s)} dL_s$

If the model is mean reverting process with Brownian motion, then the process L is Brownian motion and ϵ_k follows $N(0, 1)$. Moreover, If the model is Lévy-driven Ornstein-Uhlenbeck processes with Lévy process follows a NIG distribution, then the process L follows a NIG distribution with expectation 0 and variance 1. Thus we assume that the parameters of NIG distribution in this model are α, β, δ and μ. This means that at last, we need to estimate a set of parameters $\{m, a, s, \alpha, \beta, \delta, \mu\}$

4.2.1 Parameter Estimation procedure: step one

We estimate the subset of parameter $\{m, a, s\}$ at first using a least square method that minimize the empirical variance of the noise:

$$Var[s\epsilon] \approx \frac{1}{n}\sum_{k=0}^{n-1}(X_{k+1} - (1 + a)X_k - m)^2$$

where n is the amount of data we have.

Now he solutions to is given by:

$$\begin{bmatrix} \hat{m} \\ 1-\hat{a} \end{bmatrix} = (A'A)^{-1}A'B$$

Where A=$\begin{bmatrix} 1 & X_{n-1} \\ ... & ... \\ 1 & X_0 \end{bmatrix}$ and B=$\begin{bmatrix} X_n \\ ... \\ X_1 \end{bmatrix}$. And the estimator of s is directly followed by

$$\hat{s}^2 = \hat{s}^2 Var[\epsilon] = Var[\hat{s}\epsilon] = \frac{1}{n}\sum_{k=0}^{n-1}(X_{k+1} - (1+\hat{a})X_k - \hat{m})^2.$$

4.2.2 Parameter Estimation procedure: step two

In this step we propose a constrained maximum likelihood method to estimate the parameter $\{\alpha, \beta, \delta, \mu\}$. So far we assume that we have n+1 observations (the prices) $(X_0, X_1, ..., X_n)$ such that, for $k = 0, 1, ..., n - 1$,

$$\tilde{\epsilon}_k = X_{k+1} - (1 - \hat{a})X_k = \hat{m} + \hat{s}\epsilon_k$$

is followed by the non-centered and unnormalized NIG distribution $NIG(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\mu})$. Now, we are willing to estimate these parameters based on the likelihood function of NIG distribution.

Proposition 1: Suppose $X_1, X_2, ..., X_n \sim NIG(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\mu})$, then log-likelihood function is given by

$$nlog(\frac{\tilde{\alpha}\tilde{\delta}}{\pi}) + n\tilde{\delta}\tilde{\gamma} + \sum_{k=0}^{n-1}[\tilde{\beta}\tilde{\delta}\tau_k - \log c_k + \log K_1(\tilde{\alpha}\tilde{\delta}c_k)]$$

where $\tau_k = \frac{X_k - \tilde{\mu}}{\delta}$, $c_k = \sqrt{1+\tau_k^2}$, $\tilde{\gamma} = \sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2}$.

Proposition 2: If $X \sim NIG(\alpha, \beta, \delta, \mu)$, then for any $a \in \mathbb{R}^+$ and $b \in \mathbb{R}$, we have

$$Y = aX + b \sim NIG(\frac{\alpha}{a}, \frac{\beta}{a}, a\delta, a\mu + b)$$

Proposition 3: The first four central moments of the NIG distribution are:

$$m_1 = \mu + \delta\beta\gamma^{-1}, m_2 = \delta\alpha^2\gamma_{-3},$$
$$m_3 = 3\delta\beta\alpha^2\gamma^{-5}, m_4 = 3\delta\alpha^2(\alpha^2 + 4\beta^2)\gamma^{-7}$$

By proposition 1, we can estimate the parameters $\{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\mu}\}$ by maximize the log likelihood under constrains $\tilde{\gamma} > 0$ and $\tilde{\delta} > 0$. Due to complicated form of the density of NIG distribution, obtain a estimators for our parameters is a difficult task and so we need special numerical method to solve it.

Once the parameter set $\{\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\mu}\}$ has been estimated, we then use proposition 2. We know that

$$\epsilon_k = \frac{\tilde{\epsilon}_k}{\hat{s}} - \frac{\hat{m}}{\hat{s}} \sim NIG(\hat{s}\tilde{\alpha}, \hat{s}\tilde{\beta}, \frac{\tilde{\delta}}{\hat{s}}, \frac{\tilde{\mu} - \hat{m}}{\hat{s}}).$$

So the true estimates of the parameters $\{\alpha, \beta, \delta, \mu\}$ are:

$$\alpha = \hat{s}\tilde{\alpha}, \quad \beta = \hat{s}\tilde{\beta}, \quad \delta = \frac{\tilde{\delta}}{\hat{s}}, \quad \mu = \frac{\tilde{\mu} - \hat{m}}{\hat{s}}.$$

Recall in previous we want the expectation of ϵ_k to be 0 and variance to be 1. Thus according to proposition 3, we also need $E[\epsilon_k] = \mu + \frac{\delta\beta}{\gamma} = 0$ and $Var[\epsilon_k] = \frac{\delta\alpha^2}{\gamma^3} = 1$. Combine with the four equation above, we can conclude that we only have two free parameters (α, β).

At last, since we are using special numerical method to maximize the log-likelihood function, we need to give our method some good initial values. Under this situation, we find out the first to forth sample moments based on X_i and then solve the initial value of these parameters. Now let

$$\mu_1 = \tilde{\mu} + \tilde{\delta}\tilde{\beta}\tilde{\gamma}^{-1}, \quad \mu_2 = \tilde{\delta}\tilde{\alpha}^2\tilde{\gamma}^{-3}$$
$$\mu_3 = 3\tilde{\delta}\tilde{\beta}\tilde{\alpha}^2\tilde{\gamma}^{-5}, \quad \mu_4 = 3\tilde{\delta}\tilde{\alpha}^2\tilde{\alpha}^2 + 4\tilde{\beta}^2\tilde{\gamma}^{-7}$$

where $\mu_k = \frac{1}{n}\sum_{j=0}^{n-1}(\tilde{\epsilon_j} - \bar{X})^k$, $k = 1, 2, 3, ...$ is the kth sample moment.

Solving these four equation we obtain four initial choices of our parameters,

$$\hat{\tilde{\gamma}} = \frac{3}{\bar{S}\sqrt{3\bar{\gamma}_2 - 5\bar{\gamma_1}^2}}, \quad \hat{\tilde{\beta}} = \frac{\bar{\gamma}_1\bar{S}\hat{\tilde{\gamma}}^2}{3}$$
$$\hat{\tilde{\delta}} = \frac{\bar{S}^2\hat{\tilde{\gamma}}^3}{\hat{\tilde{\beta}}^2 + \hat{\tilde{\gamma}}^2}, \quad and \quad \hat{\tilde{\mu}} = \bar{X} - \hat{\tilde{\beta}}\frac{\hat{\tilde{\delta}}}{\hat{\tilde{\gamma}}}$$

where $\bar{X}$ and $\bar{S}$ are the sample mean and variance respectively and $\bar{\gamma}_1 = \frac{\mu_3}{\mu_2^{\frac{3}{2}}}$, $\bar{\gamma}_2 = \frac{\mu_4}{\mu_2^2} - 2$.

4.3 Heston Model

A common characteristic of energy and financial markets is the change of volatility over periods of time. Indeed, financial markets are known to have periods of stress with high volatility as can happen in a crisis when uncertainty reigns and calm periods where prices do not vary much. Energy markets are similar to that respect. Elections, changes in the national allocations of permits, discussion of a plant's shutdown or even cartel decision such as the OPEC crisis of 1973 can create temporary uncertainty in energy prices and cause panic among investors and other stakeholders. Considering the stochastic nature of the volatility, we decide to model prices using a Heston model as an alternative to the other two processes.

Definition 5: (Heston model) Under the risk-neutral probability measure Q the Heston model is given by:

$$dS(t) = rS(t)dt + \sqrt{V(t)}S(t)dW_s(t)$$
$$dV(t) = \kappa(\theta - V(t))dt + \sigma\sqrt{V(t)}dW_v(t)$$

where $W_s(t)$ and $W_v(t)$ are two Brownian motions with correlation coefficient ρ (Heston, 1993).

4.3.1 Calibration of the Heston Model

We apply the Euler-Maruyama (Begin et Al, 2014)scheme to the two equations defined above in order to discretize the process.

Algorithm 1:. Let $\hat{X}$ and $\hat{V}$ denote discrete-time approximations of X and V respectively. The Euler-Maruyama scheme applied to the above equation is given by

$$\hat{X}(h(i)) = \hat{X}(h(i-1)) + (r - \frac{1}{2}\hat{V}(h(i-1)))h$$
$$+ \sqrt{V(h(i-1))}Z_x\sqrt(h)$$

$$\hat{V}(h(i)) = \hat{V}(h(i-1)) + \kappa(\theta - V(h(i-1)))h + \sigma\sqrt{V(h(i-1))}Z_v\sqrt{(}h)$$

where Z_x and Z_y are standardized Gaussian random variables such that $corr(Z_x, Z_y) = \rho$.

As one can see, the volatility equation is a Ornstein-Uhlenbeck process, therefore we use our least squares approach to estimate its parameters. Regarding the risk-free rate r. it is arbitrarily initiated at zero. The starting value of the process is the mean of the energy price as opposed to the initial price of the time series. Since, the stock price equation does not take into account the average long term price, we concluded that using the mean as a starting point yelds better results.

5 Empirical Analysis

This section provides the main findings of this article. First, we begin by comparing the general statistic of our data with the empirical estimation of our parameters. Second, the simulations obtained are displayed. Third, the goodness of fit of the data is assessed.

5.1 Parameter Estimation

Table 1 below presents a summary of the main information regarding energy prices. Table 2 provides the estimated least squares parameter for the Lévy-pure jump and Brownian motion processes. Table 3 displays the estimates using the simulate the NIG.

The data section of this article presented the real price of each energy stock and demonstrated that they were subject to high spikes and jumps. Table 1 confirms our past impression. It is clear that prices of oil vary quite substantially with a minimum value of 17.72$/barrel and a high of 145.18$/barrel. This can potentially be explained by the power of cartels to dictate prices. Moreover, the skewness and kurtosis measurement indicate departures from the Normal as previously assumed by looking at the distribution of prices. Indeed, a positive kurtosis means that extreme events are more likely than if the data came from a Normal distribution. Consequently, we expect the flexibility provided by the NIG to improve performance with regards to a traditional Brownian motion approach.

Table 2 describes the result of three energy prices. This paper chose to focus on Energy-switching, oil, and fuel-switching as they are the main focus of this paper. κ represents the speed of mean-reversion of the process. As hinted by the plots in the previous section, we observe that mean-reversion speed of energy-switching is faster than the oil one. Additionally, we note that θ, the average price of the process, is almost equal to the actual mean. Therefore, calibrating the processes using least squares gives satisfactory results. Regarding the Heston model, parameter estimates are not presented in this section since they are hardly comparable with the other two processes, however, the simulation section presents an extensive explanation of the model.

Table 1. Summary Statistics

Energy	Energy-Switching	Oil	Coal	Natural Gas	Fuel-Switching
Mean	61.11	62.05	57.75	4.67	2.55
Median	62.17	59.38	58.02	4.03	2.97
Standard Deviation	16.20	26.89	10.11	2.25	3.98
Min	15.10	17.72	39.50	1.59	-7.16
Max	97.04	145.18	79.50	18.48	14.88
Period	14-18	00-18	10-18	00-18	14-19
Observations	223	992	425	947	1272
Skewness	-0.76	0.36	0.08	1.66	0.050
Kurtosis	0.75	-0.79	-0.47	3.94	2.45

Table 2. Least Squares Parameter Estimates for Lévy Processes

Parameter	Energy-Switching	Oil	Fuel-Switching
κ	0.1095921	0.007238064	0.008600791
θ	63.45765	64.84073	1.10466
σ	7.408024	3.064373	0.4007405

Table 3. NIG Parameter Estimates

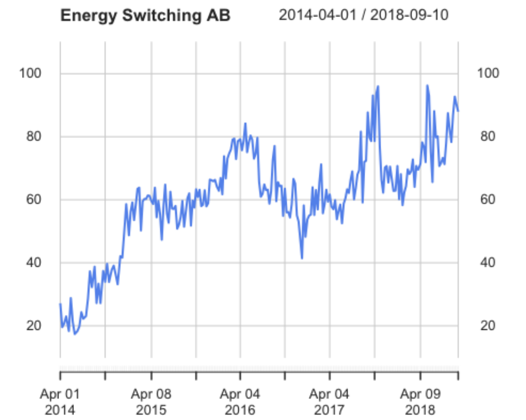
Parameter	Energy-Switching	Oil	Fuel-Switching
α	1.393878	1.099928	0.60614414
β	0.1155199	-0.1857707	-0.13472828
δ	1.059834	0.9967635	0.10518010
μ	-0.3104575	0.1708004	0.03343841

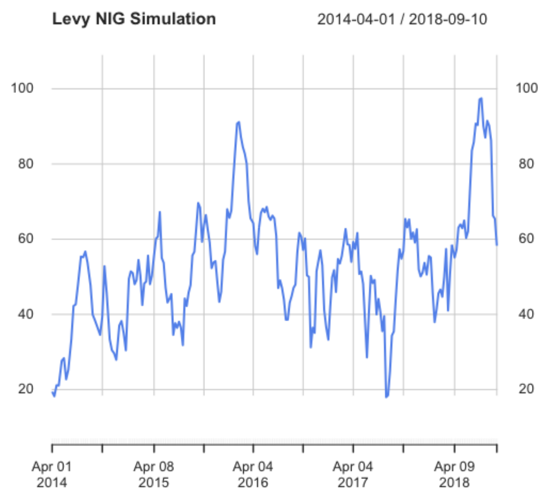
Table 4. Heston Model Parameter Estimates

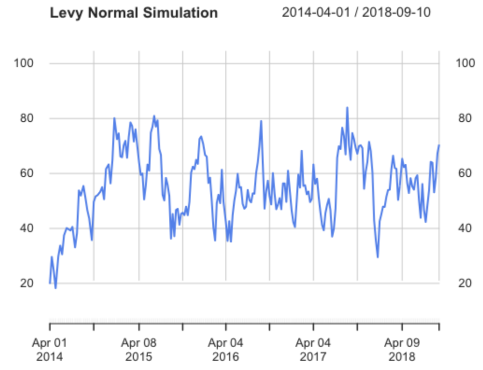
Parameter	Energy-Switching	Oil	Fuel-Switching
κ	0.5166925	0.6575034	0.2253708
θ	0.008760746	0.0004417604	1.198215
σ	0.01109988	0.0007746965	2.930622

Prices were simulated using the software *R* and various packages from the CRAN library. As an example, we choose to present the result for energy-switching (main focus) and oil (larger dataset available). As one can see in the plots that follow, the Lévy NIG process incorporates high-spike and larger volatility when compared to the Normal. If it appears clear that the NIG is better to simulate the energy-switching price, the results for oil are harder to assess. Moreover, the Heston model does not seem to fit our data correctly. A possible explanation is that the weekly volatility is not distributed as a χ^2. Therefore, the next part of this section reviews the goodness of fit test for the Normal and NIG processes, in order to confirm our visual interpretation.

Fig. 5. Simulation Results for Energy-Switching: Albertan Data, sources: NRG Stream and CanWea. Prices are expressed in CAD/MWh.







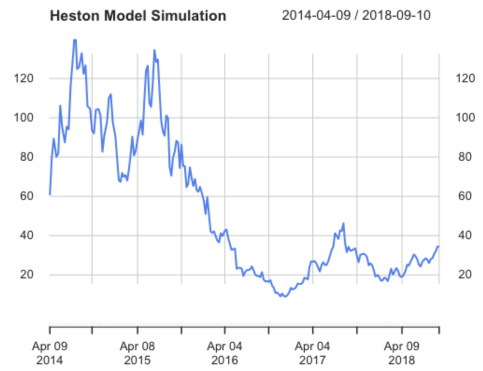
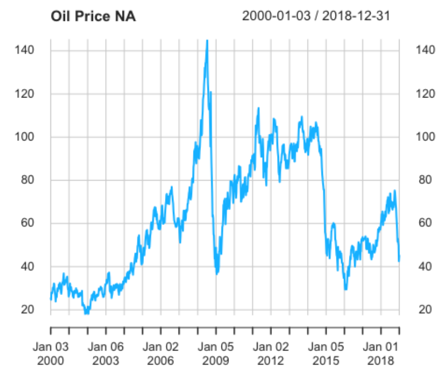
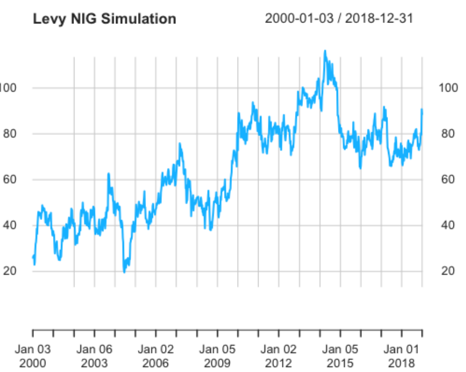
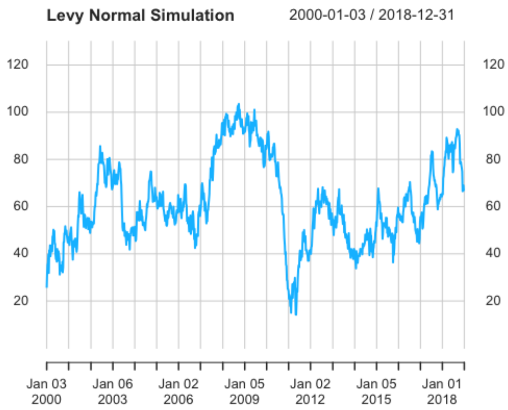
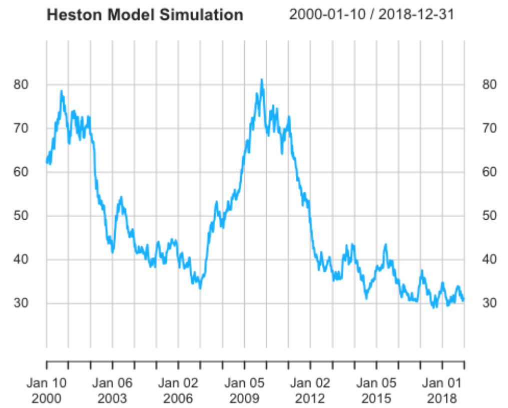


Fig. 6. Simulation Results for Oil, North American Data, source: Bloomberg. Prices are expressed in USD/barrel.









5.3 Goodness of Fit Test

The Kolmogorov-Smirnov test is a common approach to estimate the goodness of fit of the data. A p-value larger than five percent indicates that the specified model and the empirical data come from the same distribution. Table 4 gives the p-values for the selected energy prices. The p-values of the Kolmogorov-Smirnov test were calculated based on the parameter estimated and the distribution of the residuals. According to this method, both the NIG and Normal distribution are adequate to model the data. However, we cannot conclude that one method provides stronger results, even though the p-values of the NIG are higher. One must interpret the outcome of the test with caution. Indeed, the p-values appear to be quite high. Inflated p-values are not uncommon since the parameters were estimated directly using the residuals. Nonetheless, the Kolmogorov-Smirnov test remains the best approach to assess the goodness of fit of the data. We address this issue in greater details in the shortcomings. Hence, we need additional proof to show that the NIG outperforms the Normal.

First, as one can see in the Q-Q plot below for oil, the distribution of the data appears to be normal for the most part, but extreme events are more likely to happen than what the normal predicts. Therefore, we obtain better results when using a NIG approach.

Table 5. Kolmogorov-Smirnov Test

P-value	Energy-Switching	Oil	Fuel-Switching	Coal	Natural Gas
Normal	0.3677	0.8609	0.3868	0.704	0.9136
Normal Inverse Gaussian	0.4484	0.869	0.7987	0.762	0.3277

Second, the comparison of a simulated distribution residuals from a NIG and Normal compared with the actual distribution of residuals confirms this result even more. Therefore, both the Q-Qplot and histogram show that the NIG outperforms the Normal.

Fig. 7. Distribution the oil residuals

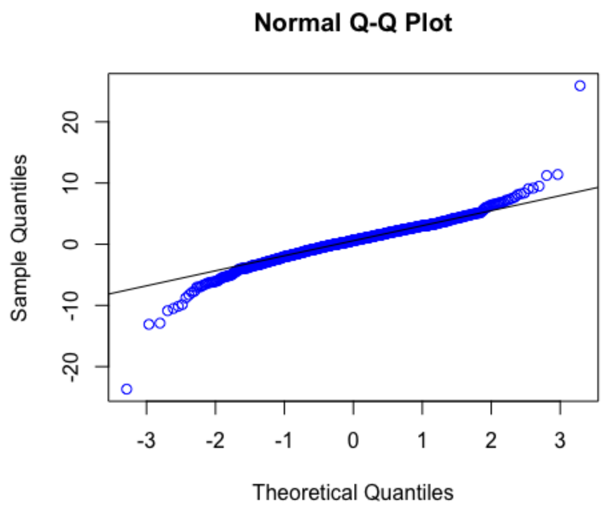
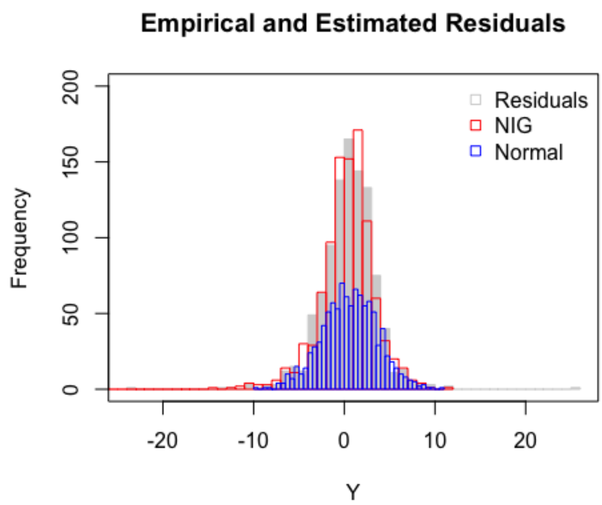


Fig. 8. Histogram of oil residuals



6 Conclusion

6.1 Summary of Main Results

The goal of this paper is to assess to review the introduction of the carbon tax and to model energy prices. The procedures employed to define the theoretical carbon price are energy-switching and fuel-switching. Furthermore, this article focused on oil, coal and natural gas separately. Three stochastic processes are considered to model prices: Lévy NIG, Lévy Normal and Heston model.

The recent changes in the coal and natural gas prices have made fuel-switching an obsolete method to price carbon. Indeed, the fuel-switching price appears to be negative for several periods. Energy-switching, on the other hand, relies on the use of renewable energies, such as wind. This approach is hard to implement due to the lack of existing infrastructure, which does not allow to switch from one energy to the other. Nonetheless, as the use of renewable increases, a policy maker could define a carbon price based on this approach. However, it should also consider alternative sources of energies, such as hydro and solar energies. Our approach is best suited for the Albertan case but might not be adequate for other provinces such as Québec, which relies mostly on hydro energy. We find that an appropriate way to tax carbon emissions, would be to tax the use of natural gas between 60 to 100 \$/MWh.

The three types of stochastic procedures yield different results. Overall, the Lévy NIG outperforms both, the Lévy Normal and Heston model. Moreover, it seems that the Heston model is not suitable for energy prices. Additionally, different frequencies could be used such as daily or even high-frequency data. We chose weekly data due to the difficulties to switch from one energy to another on a daily basis.

6.2 Discussion

This section proposes a discussion of the methodology used and the topics, which should drive further research. First, the shortcomings of the models and solutions are presented. Second, an alternative to model stochastic prices is presented. Finally, the results found are compared with the ones in the literature.

As mentioned in the previous section, the goodness of fit test of our data suffers from inflated p-values resulting from the estimation technique of our parameters. Another common approach to assess the goodness of fit is the Cramer-von Mises criterion. The idea is to compare whether the empirical distribution with the assumed distribution. The value of the test is calculated under a minimum distance estimation procedure (Anderson, 1962).

Additionally, we discovered that the choice of the risk-free rate impacted greatly the outcome of the Heston model simulation. Since energy prices depend a lot on international political decisions, it might be wrong to use the average of a 3-month t-bill interest rate issued by the federal government of the United States. Consequently, we chose to set the risk-free as zero. Further research could focus on the correct estimation of the risk-free in a world where countries have proven to default and where some countries, such as Switzerland and Denmark experience even negative interest rates on their government bonds.

In terms of renewables, future research could investigate a carbon tax based on switching from natural gas to solar or hydro energy. Our approach to use wind is best suited for the case of Alberta but might not adequate for a province such as Québec, which produces electricity using hydro power. Therefore, an optimal federal carbon price method has to take into account the different characteristics of provinces.

Another alternative could have been used to model energy prices. A Markov switching Lévy-driven Ornstein-Uhlenbeck approach, where the parameter σ can be changed under different states of a Markov chain and different state represents different economic status like inflation or a crisis, would be a potential candidate to obtain better results. Hence, future research should try and implement this type of process.

The findings of this paper are similar to the results of Goutte and Chevalier (2015), who investigated the fuel-switching price on the European market from 2007 to 2010. At that time, the price of coal was relatively cheap in comparison to the price of natural gas. Their main results indicate that the Lévy NIG outperforms the Normal by far. Moreover, they considered the case of a Poisson process and showed it was not suitable to model energy prices.

Acknowledgment

The authors would like to thank the NSERC and University of Calgary for providing the necessary funding to conduct this research. We are also grateful to the International Association for Energy Economics (IAEE) and its member for allowing us to present our research during the 42nd International conference on *Local Energy, Global Markets* held at HEC Montréal May 29th to June 1st.

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